

SIGNATURE _____ NAME _____

Student ID # _____

**Physics 410
Fall 2014
Prof. Anlage
FINAL Exam
16 December, 2014
CLOSED BOOK, Calculator Permitted**

Point totals are given for each part of the question.

If you run out of room, continue writing on the back of the same page. If you do so,
make a note on the front part of the page!

Note: You must solve the problem following the instructions given in the problem.
Correct answers alone will not receive full credit.

Partial Credit:

→ Show Your Work! Answers written with no explanation will not receive full credit.

→ You can receive credit for describing the method you would use to solve a problem, even if you missed an earlier part.

Problem	Credit	Max. Credit
1		25
2		25
3		25
4		25
TOTAL		100

$$\vec{r} \cdot \vec{s} = rs \cos \theta \quad \vec{r} \times \vec{s} = \det \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ r_x & r_y & r_z \\ s_x & s_y & s_z \end{bmatrix} \quad \vec{F} = m\ddot{\vec{r}} \quad \text{Constant } a: x(t) = x_0 + v_0 t +$$

$$\frac{1}{2}at^2; v(t) = v_0 + at; v_f^2 - v_i^2 = 2a\Delta x \quad \vec{f} = -f(v)\hat{v} \quad f(v) = bv + cv^2$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad m\dot{v} = -\dot{m}v_{ex} + F^{ext} \quad v - v_0 = v_{ex} \ln \frac{m_0}{m} \quad \vec{R} = \frac{1}{M} \sum_{\alpha=1}^N m_{\alpha} \vec{r}_{\alpha}$$

$$\vec{R} = \frac{1}{M} \int \vec{r} \, dm = \frac{1}{M} \int \rho \, \vec{r} \, dV \quad \vec{\ell} = \vec{r} \times \vec{p} \quad \vec{L} = \sum_{\alpha=1}^N \vec{r}_{\alpha} \times \vec{p}_{\alpha} \quad \vec{L} = \vec{L}^{ext} \quad \Delta T =$$

$$T_2 - T_1 = \int_1^2 \vec{F} \cdot d\vec{r} = W(1 \rightarrow 2) \quad T = mv^2/2 \quad U(\vec{r}) = -W(\vec{r}_0 \rightarrow \vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{F}(\vec{r}') \cdot d\vec{r}'$$

$$\vec{\nabla} \times \vec{F} = 0 \quad \vec{F} = -\vec{\nabla} U \quad E = T + U_1 + \dots + U_n \quad \Delta E = W_{nc} \quad t = \sqrt{\frac{m}{2}} \int_{x_0}^x \frac{dx'}{\sqrt{E - U(x')}} \\ \vec{F}(\vec{r}) = f(\vec{r})\hat{r} \quad U = U^{int} + U^{ext} = \sum_{\alpha} \sum_{\beta > \alpha} U_{\alpha\beta} + \sum_{\alpha} U_{\alpha}^{ext} \quad \text{Net force on particle } \alpha =$$

$$-\nabla_{\alpha} U \quad T + U = \text{constant} \quad F = -kx \leftrightarrow U = \frac{1}{2}kx^2 \quad \ddot{x} = -\omega^2 x \leftrightarrow x(t) = A \cos(\omega t - \delta)$$

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0 \leftrightarrow x(t) = Ae^{-\beta t} \cos(\omega_1 t - \delta) \quad (\text{assuming } \beta < \omega_0), \beta = \frac{2b}{m}, \text{damping force} = -bv, \omega_0 = \sqrt{\frac{k}{m}}, \omega_1 = \sqrt{\omega_0^2 - \beta^2} \quad F(t) = mf_0 \cos(\omega t), x(t) =$$

$$A \cos(\omega t - \delta), \text{ where } A^2 = \frac{f_0^2}{(\omega_0^2 - \omega^2)^2 + 4\beta^2 \omega^2} \quad \delta = \tan^{-1} \left(\frac{2\beta\omega}{\omega_0^2 - \omega^2} \right) \quad S =$$

$$\int_{x_1}^{x_2} f[y(x), y'(x), x] dx, \quad \frac{\partial f}{\partial y} - \frac{d}{dx} \frac{\partial f}{\partial y'} = 0$$

$$S = \int_{u_1}^{u_2} f[x(u), y(u), x'(u), y'(u), u] du, \quad \frac{\partial f}{\partial x} = \frac{d}{du} \frac{\partial f}{\partial x'}, \text{ and } \frac{\partial f}{\partial y} = \frac{d}{du} \frac{\partial f}{\partial y'} \quad \mathcal{L} = T - U$$

$$\frac{\partial \mathcal{L}}{\partial q_i} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}_i} \quad [i = 1, \dots, n] \quad p_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i} \quad \mathcal{H} = \sum_{i=1}^n p_i \dot{q}_i - \mathcal{L} \quad \vec{r} = \vec{r}_1 - \vec{r}_2 \quad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$U_{eff} = U(r) + U_{cf}(r) = U(r) + \frac{\ell^2}{2\mu r^2} \quad u''(\varphi) = -u(\varphi) - \frac{\mu}{\ell^2 u(\varphi)^2} F$$

$$r(\varphi) = \frac{c}{1 + \epsilon \cos \varphi} \text{ for } F = -\frac{\gamma}{r^2}, \text{ with } c = \frac{\ell^2}{\gamma \mu} \quad E = \frac{\gamma^2 \mu}{2 \ell^2} (\epsilon^2 - 1)$$

$$\vec{F}_{inertial} = -m\vec{A} \quad \vec{\omega} = \omega \hat{u} \quad \vec{v} = \vec{\omega} \times \vec{r} \quad \left(\frac{d\vec{Q}}{dt} \right)_{S_0} = \left(\frac{d\vec{Q}}{dt} \right)_S + \vec{\Omega} \times \vec{Q}$$

$$m\ddot{\vec{r}} = \vec{F} + \vec{F}_{cor} + \vec{F}_{cf}, \text{ with } \vec{F}_{cor} = 2m\dot{\vec{r}} \times \vec{\Omega}, \text{ and } \vec{F}_{cf} = m(\vec{\Omega} \times \vec{r}) \times \vec{\Omega}$$

$$\vec{g} = \vec{g}_0 + (\vec{\Omega} \times \vec{R}) \times \vec{\Omega} \quad N_{oc} = N_{inc} n_{tar} \sigma_{oc} \quad N_{sc}(\text{into } d\Omega) = N_{inc} n_{tar} \frac{d\sigma}{d\Omega}(\theta, \phi) d\Omega$$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin \theta} \left| \frac{db}{d\theta} \right| \quad \frac{d\sigma}{d\Omega} = \left(\frac{kqQ}{4E \sin^2(\theta/2)} \right)^2 \quad \dot{q}_i = \partial \mathcal{H} / \partial p_i \text{ and } \dot{p}_i = -\frac{\partial \mathcal{H}}{\partial q_i} \quad [i = 1, \dots, n]$$

$$\vec{L} = \vec{L}(\text{motion of CM}) + \vec{L}(\text{motion relative to CM})$$

$$T = T(\text{motion of CM}) + T(\text{motion relative to CM}) \quad \vec{L} = \vec{I} \vec{\omega}$$

$$\vec{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \quad I_{xx} = \sum_{\alpha} m_{\alpha} (y_{\alpha}^2 + z_{\alpha}^2), \text{ etc.} \quad I_{xy} = -\sum_{\alpha} m_{\alpha} x_{\alpha} y_{\alpha}, \text{ etc.} \quad \vec{L} = \lambda \vec{\omega}$$

$$(\vec{I} - \lambda \vec{1}) \vec{\omega} = 0 \quad \text{Characteristic equation: } \det(\vec{I} - \lambda \vec{1}) = 0 \quad \vec{I}' = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \quad \dot{\vec{L}} + \vec{\omega} \times \vec{L} =$$

$$\vec{\Gamma} \quad \vec{M} \ddot{\vec{q}} = -\vec{K} \vec{q} \quad T = \frac{1}{2} \sum_{j,k} M_{jk} \dot{q}_j \dot{q}_k \quad U = \frac{1}{2} \sum_{j,k} K_{jk} q_j q_k \quad \vec{q}(t) = \text{Re}(\vec{a} e^{i\omega t})$$

$$(\bar{K} - \omega^2 \bar{M})\vec{a} = 0 \quad \ddot{\phi} + 2\beta\dot{\phi} + \omega_0^2 \sin \phi = \gamma\omega_0^2 \cos(\omega t), \text{ with } \gamma = \frac{F_0}{mg}, \text{ and } F(t) = F_0 \cos(\omega t) \quad |\Delta\phi(t)| \sim K e^{\lambda t} \quad \dot{q}_i = \partial\mathcal{H}/\partial p_i \text{ and } \dot{p}_i = -\frac{\partial\mathcal{H}}{\partial q_i} \quad [i = 1, \dots, n]$$

$$\Delta t = \gamma \Delta t_0 \quad \gamma = 1/\sqrt{1-\beta^2}, \quad \beta = V/c \quad \ell = \ell_0/\gamma \quad x' = \gamma(x-Vt), \quad y' = y, \quad z' = z, \\ t' = \gamma(t-Vx/c^2) \quad v'_x = \frac{v_x-V}{1-v_xV/c^2}, \quad v'_y = \frac{v_y}{\gamma(1-v_xV/c^2)}, \quad v'_z = \frac{v_z}{\gamma(1-v_xV/c^2)} \quad q^{(4)'} = \Lambda q^{(4)} \\ x^{(4)} \cdot y^{(4)} = x_1y_1 + x_2y_2 + x_3y_3 - x_4y_4 \quad u^{(4)} = \frac{dx^{(4)}}{dt_0} = \gamma(\vec{v}, c) \quad p^{(4)} = mu^{(4)} = \\ (\gamma m \vec{v}, \gamma mc) = (\vec{p}, E/c) \quad \vec{\beta} = \vec{p}c/E \quad p^{(4)} \cdot p^{(4)} = -(mc)^2 \quad E = \gamma mc^2 \quad E^2 = (mc^2)^2 + \\ (\vec{p}c)^2 \quad \vec{F} = d\vec{p}/dt \text{ with } \vec{p} = \gamma m \vec{v} \quad K^{(4)} = \frac{dp^{(4)}}{dt_0} = \gamma(\vec{F}, \vec{v} \cdot \vec{F}/c)$$

$$\text{Photon: } p^{(4)}_\gamma = \frac{\hbar\omega}{c}(\hat{k},1)$$

$$\text{Binomial expansion for } x \ll 1: (1+x)^n \cong 1 + nx + \frac{n(n-1)}{2}x^2$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots \quad (x < 1)$$